COLLISION EFFICIENCY OF NEARLY EQUAL CLOUD DROPS

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ABSTRACT

Recomputation of the collision efficiency of nearly equal water drops falling in air gave values which increase as the size ratio approaches unity for radii of the bigger drop larger than 63 μ, and decreased as though tending toward zero for smaller radii. For the bigger radii the new computed values are consistent with experimental results, but for radii around 40 μ they are not.

1. INTRODUCTION

The values of collision efficiency of pairs of spheres falling in a viscous medium computed by Shafrir and Neiburger [5, 6] agree fairly well with values obtained experimentally for unequal but similarly sized spheres (Neiburger and Pruppacher [2], Neiburger [1]). However, for nearly equal spheres the original computations indicated zero efficiency, whereas the experiments of Telford et al. [8], for 77-μ radius water drops falling in air, had given a value not merely non-zero but considerably larger than the geometric value, and recently Woods [9] and Woods and Mason [10] carried out experiments which also gave non-zero values for equal drops with radii in the range 35 to 95 μ.

This discrepancy between computed and experimental values for nearly equal drops was attributed in part to the fact that the computations were carried out with large initial separations of the drops (100 radii) while in the experiments they were initially much closer together. At large separations the terminal velocities of nearly equal drops would be nearly the same. The velocity of the upper drop relative to the lower one would be very small and the time to overtake correspondingly long. During the many time steps required in the machine computation in this case computational error could accumulate and be responsible for the inconsistency between the computations and the experiments.

On the other hand this long period of overtake might provide a physical reason to expect the collision efficiency to be zero. The deflection of the lower sphere out of the path of the upper is caused by the viscous drag on the former by the fluid passing around the latter. When the period of overtake is long the drag force has plenty of time to act to carry the lower drop out of the path of the upper. However the flow around the lower sphere exerts a similar drag on the upper drop, deflecting it in the same direction. If the deflection of the lower drop is greater the collision efficiently will be small or zero. If the deflection of the upper drop is greater the efficiency may be greater than the geometric. Pearcey and Hill [3] considered that for large drops (Reynolds number $Re \geq 1$) the asymmetry of the flow, with the upper of two nearly equal drops acted on by the wake of the lower, would lead to larger deflection of the upper drop and large collision efficiencies, in accord with the results of their computations. When our computations gave opposite results we judged that they had overestimated the wake effect, a conclusion fortified by evidence that they had made an error in computing the flow field (Shafrir [4]).

In addition to the possibility of accumulation of computational error, the computation involved assumptions and simplifications which might lead to erroneous results. It was interesting, therefore, to see whether more refined computational procedures and reduction of the initial separation would result in better agreement between theoretical and experimental values.

It should be pointed out, incidentally, that while the experiments were carried out with ingenuity and care, the possibility that their results were in error even for the small initial separations cannot be excluded. In both sets of experiments streams of drops were introduced rather than isolated pairs, and the behavior of each drop was influenced by all the others. Other disturbing factors, such as turbulence and wall effects, could have entered also. However, our premise was that the experimental results were probably correct, and we sought to improve the method of computation to minimize the errors introduced thereby.

2. METHOD OF COMPUTATION

The collision efficiency is defined by considering a cloud of drops of radius a through which a larger drop of radius A is falling. If the a drops were not deflected by the motion of the air around the A drop, all the a drops with centers within a distance $a+A$ of the vertical downward through the center of the A drop would collide with it. Because the drops are deflected, only those within a distance R from the vertical actually collide. The ratio of the drops that would actually collide to those that would be expected from geometric considerations to collide is the collision efficiency E. Thus

$$E=\frac{R^2}{(a+A)^2}. \tag{1}$$

Alternatively the collision efficiency is sometimes given as $E'=R^2/A^2$.

It is convenient to let $a/A=p$ and define a *linear collision efficiency*

$$y_c=R/A. \tag{2}$$

Then

$$\left.\begin{aligned} E&=y_c^2/(1+p)^2 \\ E'&=y_c^2 \end{aligned}\right\}. \tag{3}$$

The cases under discussion are those for which $p\approx 1$. The original computations could be carried out only for p slightly greater than 0.9. The velocities of the drops were too nearly equal for larger p values to permit completion of the relative trajectories.

The computational program has been refined subsequently in connection with experimental tests of the basic method of computing the collision efficiencies. The essential physical background of the computation of the trajectories was not changed, but the integration procedure was made more efficient and a new method of cutting and testing the time steps was introduced in order to limit the total error. To save machine time on the IBM 7094 some of the subroutines for computation of the trajectories of the drops were expressed in machine language instead of Fortran. A criterion for the required accuracy of each time step as a fraction of the total time of overtake was developed in order to limit to 0.01 A the total error of positions at the time the A drop caught up with the a drop. Every sixth time step until the drops were 10 A apart, and then every second time step the computation was repeated using two one-half sized time steps, and if the difference exceeded the criterion the time step was halved and the process repeated. By this method it is estimated that the finite difference computation contributed no more than 2 percent to the error in y_c.

For details of the approximations made in computing the trajectories and evaluating y_c the reader is referred to the aforementioned publications of Shafrir and Neiburger. Briefly, each drop was treated as acted on jointly by gravity and by the drag force which it would experience if the air were moving with the velocity the other drop would induce at its center in its absence. This procedure was used by Pearcey and Hill [3], but whereas they used solutions to Oseen's linearized equations to obtain the fluid velocity we used exact numerical solutions to the complete nonlinear equations.

3. RESULTS

The refined computations were carried out for the drop sizes previously used and for $0.8\leq p<1.0$. The computations could not be carried out for exactly equal drops.

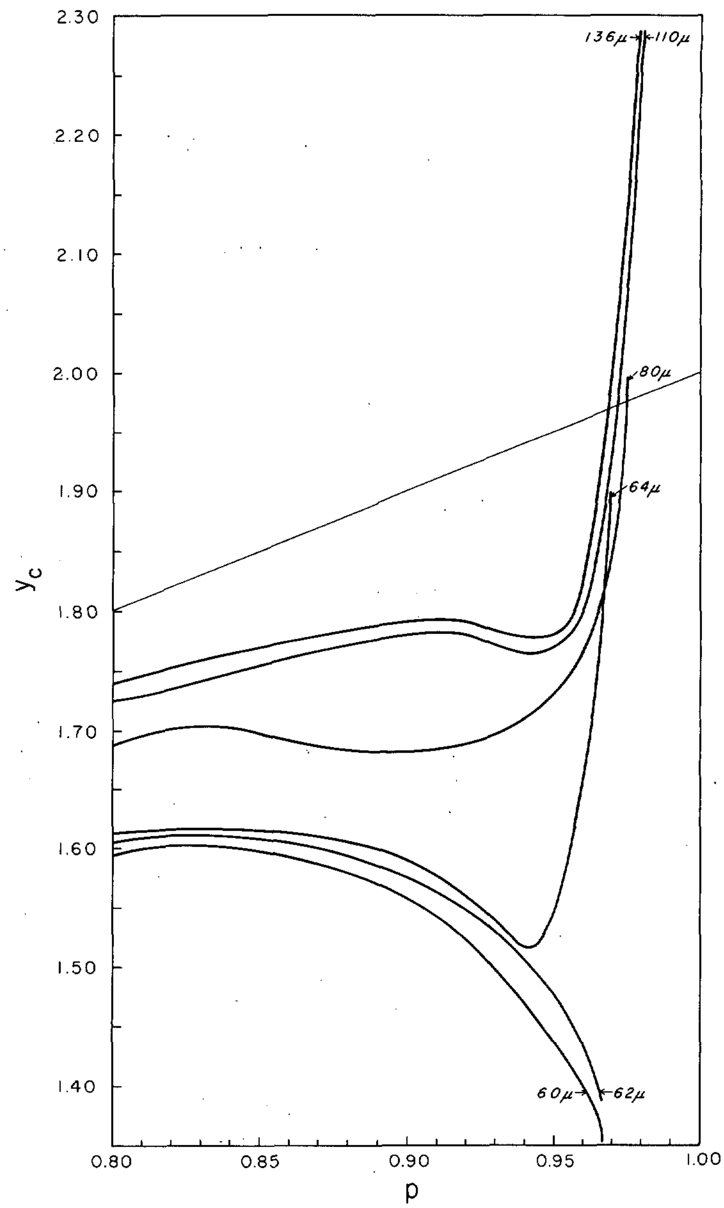

FIGURE 1.—Curves representing the linear collision efficiency y_c as a function of the ratio of drop radii p for various radii of the larger drop. The sloping straight line represents the geometrical linear collision efficiency $y_c=1+p$. The labels on the right end of each curve give the large radius A to which it refers.

Figure 1 shows the results for $A=60\mu$, 62μ, 64μ, 80μ, 110μ, and 136μ.

For the 60-μ and 62-μ drops the collision efficiency dropped as p approached unity, as it had in the results of the earlier computations. However for the larger drops, beginning with 64μ the linear collision efficiency has a minimum at a value of p greater than 0.85 and then increases for larger values of p. For $A\geq 80\mu$ the computed y_c values for the largest p exceeded the geometric value, $1+p$. It is reasonable to assume that as p approaches closer to unity, y_c would increase still further.

The value found experimentally by Telford et al. for 77-μ drops ($E'=12.6\pm 3.4$) corresponds to $y_c=3.5\pm 0.5$.

For 80-μ drops and $p=0.975$ the computed value was 1.99. While we have no basis for extrapolating the computed value to $p=1.0$, the slope of the last part of the 80-μ curve as it is drawn in figure 1 would give a value at $p=1.0$ of 3.25. One can say, therefore, that the computed values are not inconsistent with Telford's experimental result.

Table 1 gives the data from Woods' experimental results pertinent to the computed values and the computed y_c for the largest size ratio p for which the computation was carried out.

The values for E found by Woods, and thus the y_c values derived from them and shown in the table, were based on the largest initial separation of groups of eight to 36 observations. They are thus lower bounds on these quantities. Woods states that "for a set of 20 observations there is an 88 percent probability that the tabulated values lie within 10 percent of those that would be given by an infinite set of observations." Except for the 35–45-μ group the experimental values seem fairly consistent with the computed values. In the 75–85-μ group the experimental value is slightly lower than the value computed for $A=80$ and $p=0.98$. A still higher value would be expected for computation with $p=1.0$, but since in this group there were only eight experimental observations the experimental result would be expected to be low. For the 65–75 group, experimental value is the same as computed for $p=0.96$–0.97, so that it too appears to be low; in this group there were 36 observations so that the discrepancy cannot be attributed to an insufficient number of data.

The 55–65-μ group straddles the separation between the computed values which decrease as p approaches unity and those which increase. The computed and experimental values would be consistent if the instance of highest initial separation, on which the $y_c=1.8$ is based, involved collision of 64-μ drops. Since few of the 34 collisions in that group would have been 64-μ drops the fact that the experimental value for $p=1$ is less than the computed value for $p=0.97$ is reasonable. However, Woods shows examples of collisions between equal 62-μ drops, so that even if the computations are correct in showing the collision efficiency for 62-μ drops decreasing for increasing p the experiments show that the value would not be zero for $p=1$.

TABLE 1.—*Comparison and experimental and computed collision efficiencies*

Woods' experiment: $p=1$				Present computation		
$A(\mu)$	E	y_c	No. of events	$A(\mu)$	p	y_c
35–45	0.5	1.4	14	40	0.80	1.36
				40	0.90	1.21
45–55	0.7	1.7	30			
55–65	0.85	1.8	34	60	0.80	1.59
				60	0.97	1.35
				64	0.97	1.89
65–75	0.9	1.9	36	66	0.97	1.92
				69	0.96	1.86
75–85	0.95	1.95	8	78	0.96	1.75
				80	0.98	1.99
				82	0.98	2.03

It is the 35–45-μ group for which the computations are in complete disagreement with the experimental results. The computed values decrease with p increasing from 0.8 to 0.9. Because velocity fields for flow around spheres were not available for drops from 37- to 39-μ radius computations could not be carried out for $p>0.9$, but it is logical to suppose that computed values of y_c would continue to decrease, perhaps to zero, as p approaches one. The experimental results could be consistent with the computations if among the 14 events in this experimental group some were between unequal drops for which p was as small as 0.8. However, Woods states in his thesis that the drops were closely equal, with the size varying not more than $\pm\frac{1}{2}\mu$.

Woods states further that equal drops with $A<35\mu$ were never observed to collide, and that the occasional collisions that occurred for 35–40-μ radius drops were associated with turbulence in the ambient air. He concludes that the boundary between zero and large collision efficiency for equal drops is 40-μ radius. The computations indicate this separation to be at $A=63\mu$.

One other experimental evidence is available on this point. Telford and Thorndike [7] carried out experiments in which nearly equal drops with diameter about 45μ were observed to collide and coalesce, while smaller (35-μ diameter) drops did not. These experiments would place the equal drop collision boundary at about $A=20\mu$, rather than Woods' $A=40\mu$ or the computed $A=63\mu$.

It may be mentioned, incidentally, that Telford and Thorndike considered that their observations "strongly support the theoretical treatment made by Hocking." However, for collision of nearly equal drops Hocking's computations predicted zero efficiency for 30-μ radius as well as 20-μ or smaller. It may be that they were thinking of Hocking's conclusion that for $A<19\mu$ the efficiency is zero for all values of p, but this was not tested in their experiment.

Pearcey and Hill's computations gave collision efficiencies that increased with p, but they differ from the present computations in that very large collision efficiencies are predicted for nearly equal drops of all sizes, with the geometric efficiency exceeded even for 14-μ radius drops. For larger radii they are extremely large. Table 2 gives some values estimated from their diagram. It is quite evident that Pearcey and Hill's collision effi-

TABLE 2.—*Linear collision efficiencies from Pearcey and Hill's computation*

A	y_c		
	$p=0.98$	$p=0.99$	$p=0.999$
14	1.4	1.6	2.6
19	2.0	2.7	6.5
31	3.5	4.7	14
43	4.9	7.1	24
73	6.9	10.5	41
106	8.5	14	59

ciencies for nearly equal spheres are much larger than those suggested by the experiments, even those of Telford et al. [8].

4. CONCLUSIONS

The refined computations show that nearly equal drops which are sufficiently large have large collision efficiencies. While they exceed the geometric limit, they are not as large as those computed by Pearcey and Hill. The experimental results are consistent with the new computations except with respect to the limiting size between drops for which the efficiency decreases as equality is approached and those for which it increases.

Whether Woods' limit of 40-μ radius or the computed limit of 63μ is correct, the implications with respect to formation of rain by the coalescence process is the same. Since condensation does not produce monodisperse clouds with such large drops the high collision efficiency for equal drops of sizes larger than these would play no role in initiating precipitation. Normally there would be a spectrum of drop sizes and before the largest of them approached this size they would start colliding and coalescing with smaller drops for which they attain non-zero collision efficiencies earlier. Even when the drops exceed the size for which equal drops can collide it would remain a negligible factor in the rate of drop growth, for it is highly improbable that the drop immediately below a given drop would be approximately the same size.

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